

## DIFFERENTIAL GEOMETRY II FINAL EXAMINATION

Total marks: 60

Attempt all questions

Time: 3 hours (10 am - 1 pm)

- (1) Calculate the exponential of the  $2 \times 2$  matrix

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

. (5 marks)

- (2) Show that the tangent bundle of any Lie group is trivial. (10 marks)  
(3) Compute the Lie algebra of  $SU(n)$ . (10 marks)  
(4) Prove Stokes Theorem for the closed upper half plane  $\mathbb{H}^2$ . (10 marks)  
(5) Prove that the Mobius strip is not an orientable manifold, and the 2-sphere  $S^2$  is an orientable manifold. (5+5=10 marks)  
(6) Show that any smooth manifold has a Riemannian metric. Define the Levi-Civita connection and the curvature (of the connection) on a Riemannian manifold. What is the Levi-Civita connection of a smooth surface in  $\mathbb{R}^3$ ? (5+5+5 = 15 marks)